

## Corrections

### ENGINEERING

Correction for “Lipopeptide nanoparticles for potent and selective siRNA delivery in rodents and nonhuman primates,” by Yizhou Dong, Kevin T. Love, J. Robert Dorkin, Sasilada Sirirungruang, Yunlong Zhang, Delai Chen, Roman L. Bogorad, Hao Yin, Yi Chen, Arturo J. Vegas, Christopher A. Alabi, Gaurav Sahay, Karsten T. Olejnik, Weiheng Wang, Avi Schroeder, Abigail K. R. Lytton-Jean, Daniel J. Siegwart, Akin Akinc, Carmen Barnes, Scott A. Barros, Mary Carioto, Kevin Fitzgerald, Julia Hettinger, Varun Kumar, Tatiana I. Novobrantseva, June Qin, William Querbes, Victor Koteliensky, Robert Langer, and Daniel G. Anderson, which appeared in issue 11, March 18, 2014, of *Proc Natl Acad Sci USA* (111:3955–3960; first published February 10, 2014; 10.1073/pnas.1322937111).

The authors note that they omitted additional funding sources in the Acknowledgments. The corrected Acknowledgments section should instead appear as “The authors thank the Koch Institute Swanson Biotechnology Center for technical support, specifically Flow Cytometry Core. This work was supported by the National Cancer Institute Center of Cancer Nanotechnology Excellence at MIT-Harvard (U54-CA151884), the National Heart, Lung, and Blood Institute, National Institutes of Health (NIH), as a Program of Excellence in Nanotechnology (PEN) Award, Contract #HHSN268201000045C, as well as by Alnylam Pharmaceuticals and the NIH Grants R01-EB000244–27, 5-R01-CA132091–04, and R01-DE016516–03. Y.D. acknowledges support from the National Institute of Biomedical Imaging and Bioengineering for his postdoctoral fellowship 1F32EB017625.”

[www.pnas.org/cgi/doi/10.1073/pnas.1404711111](http://www.pnas.org/cgi/doi/10.1073/pnas.1404711111)

### PHYSICS

Correction for “Testing Turing’s theory of morphogenesis in chemical cells,” by Nathan Tompkins, Ning Li, Camille Girabawe, Michael Heymann, G. Bard Ermentrout, Irving R. Epstein, and Seth Fraden, which appeared in issue 12, March 25, 2014, of *Proc Natl Acad Sci USA* (111:4397–4402; first published March 10, 2014; 10.1073/pnas.1322005111).

The authors note that Nathan Tompkins and Ning Li contributed equally to this work.

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### BIOCHEMISTRY

Correction for “Overexpressing mouse model demonstrates the protective role of Muc5ac in the lungs,” by Camille Ehre, Erin N. Worthington, Rachael M. Liesman, Barbara R. Grubb, Diane Barbier, Wanda K. O’Neal, Jean-Michel Sallenave, Raymond J. Pickles, and Richard C. Boucher, which appeared in issue 41, October 9, 2012, of *Proc Natl Acad Sci USA* (109:16528–16533; first published September 24, 2012; 10.1073/pnas.1206552109).

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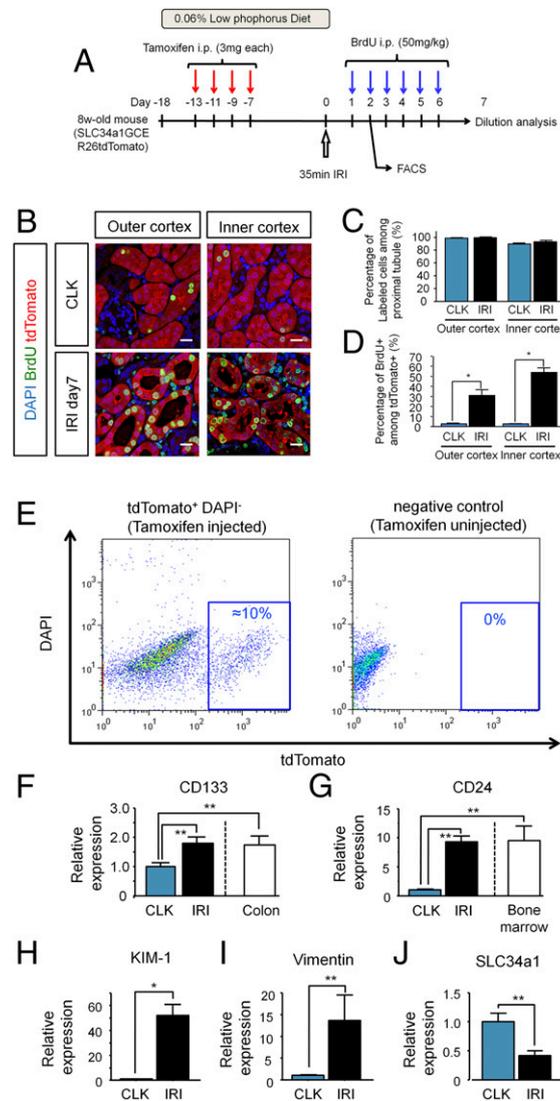
[www.pnas.org/cgi/doi/10.1073/pnas.1405175111](http://www.pnas.org/cgi/doi/10.1073/pnas.1405175111)

**MEDICAL SCIENCES**

Correction for “Differentiated kidney epithelial cells repair injured proximal tubule,” by Tetsuro Kusaba, Matthew Lalli, Rafael Kramann, Akio Kobayashi, and Benjamin D. Humphreys, which appeared in issue 4, January 28, 2014, of *Proc Natl Acad*

*Sci USA* (111:1527–1532; first published October 14, 2013; 10.1073/pnas.1310653110).

The authors note that Fig. 3 appeared incorrectly. The corrected figure and its legend appear below.



**Fig. 3.** Fully differentiated cells accomplish proximal tubular repair after acute injury. (A) Experimental scheme. *SLC34a1*<sup>GCE</sup>; R26tdTomato mice were placed on a low-phosphorus diet (0.06%) 5 d before tamoxifen administration. After they returned to a normal diet, severe IRI was performed at day 0, with BrdU administration daily for 7 d, after which the mice were killed. (B–D) Proximal tubules were labeled efficiently, and only a few cells incorporated BrdU in uninjured CLK. In IRI kidneys, there was substantial BrdU incorporation in tdTomato+ proximal tubular cells without dilution of label upon quantitation. (E) Demonstration of the gating strategy for the FACS of tdTomato-positive cells. Kidney cell suspensions of mice not injected with tamoxifen served as a negative control (Right). (F–J) Quantitative PCR of tdTomato-positive cells showed that CD133 (F), CD24 (G), vimentin (H), and KIM-1 (I) mRNA levels were substantially increased in tdTomato-positive cells in injured kidney. In addition, *SLC34a1* mRNA level decreased in injured kidney (J), indicating injured, fully differentiated tubular epithelial cells lose a differentiated phenotype after injury. Bone marrow cells serve as a positive control for stem cell markers. Average ± SE, \**P* < 0.01, \*\**P* < 0.05. (Scale bars, 15 μm.)

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# Testing Turing's theory of morphogenesis in chemical cells

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**Alan Turing, in "The Chemical Basis of Morphogenesis" [Turing AM (1952) *Philos Trans R Soc Lond* 237(641):37–72], described how, in circular arrays of identical biological cells, diffusion can interact with chemical reactions to generate up to six periodic spatiotemporal chemical structures. Turing proposed that one of these structures, a stationary pattern with a chemically determined wavelength, is responsible for differentiation. We quantitatively test Turing's ideas in a cellular chemical system consisting of an emulsion of aqueous droplets containing the Belousov–Zhabotinsky oscillatory chemical reactants, dispersed in oil, and demonstrate that reaction-diffusion processes lead to chemical differentiation, which drives physical morphogenesis in chemical cells. We observe five of the six structures predicted by Turing. In 2D hexagonal arrays, a seventh structure emerges, incompatible with Turing's original model, which we explain by modifying the theory to include heterogeneity.**

pattern formation | nonlinear dynamics | chemical oscillations | chemical dynamics

The Turing model of morphogenesis offers an explanation for how identical biological cells differentiate and change shape (1). It is difficult to overstate the impact Turing's model has had on developmental biology and the broad field of reaction-diffusion systems (2–9). The Turing model consists of two cases: The first, applicable for a ring of continuous material, has been experimentally confirmed in chemical systems (10–14). The second case, relevant to biology, consists of a ring of discrete cells, each of which contains interacting chemical species that can diffuse to neighboring cells through a chemical selective membrane. However, as the two theories for the cases are different, establishing the Turing model for continuous systems does not prove that the model holds when the chemistry is compartmentalized. Due to challenges in microfabrication, the case of a ring of cells has not previously been experimentally tested in chemical systems. In biology, where networks of cells arise naturally, the Turing model remains controversial because comparison of experiment and theory is hampered by incomplete knowledge of the morphogens involved in development, the rate constants of the reactions, the mechanisms of intercellular coupling, and the role of elasticity (5, 7, 15, 16).

We report an experimental reaction-diffusion system ideally suited for testing Turing's ideas in synthetic "cells" consisting of microfluidically produced surfactant-stabilized emulsions (17, 18) in which aqueous droplets containing the Belousov–Zhabotinsky (BZ) oscillatory chemical reactants (19) are dispersed in oil. In contrast to biology, here the chemistry is understood, rate constants are known, and interdrop coupling is purely diffusive. We explore a large set of parameters through control of concentrations, drop size, spacing, and spatial arrangement of the drops in lines and rings in one dimension and hexagonal arrays in two dimensions. Quantitative comparison of theory and experiment reveals two surprises: A structure not predicted by Turing's analysis is observed, and we measure coupling strengths orders of magnitude weaker than predicted. Nevertheless, in the majority

of cases, we find Turing's model to be exceedingly accurate. Most significantly, we experimentally establish Turing's prediction that interacting identical cells differentiate into chemically distinct populations, which subsequently transform physically in size, thereby demonstrating that these synthetic cells are pluripotent and that abiotic materials can undergo morphogenesis via the Turing mechanism. For one-dimensional arrays of drops, we observe six distinct spatiotemporal patterns, all of which are predicted by the Turing model. In closed-packed 2D arrays, we observe an additional pattern, of a mixed spatial-temporal nature that is incompatible with Turing's original model. We develop a theory, capable of describing this mixed pattern, which posits that the pattern arises from nonlinearity coupled with slight heterogeneity in cellular chemistry and/or coupling strength. As our theory is generic, and heterogeneity is ubiquitous in nature, we expect this pattern to occur in a wide range of reaction-diffusion systems.

The BZ reaction (19), the metal ion-catalyzed oscillatory oxidation of an organic substrate, typically malonic acid (MA), by acidic bromate, has become the prototype of nonlinear dynamics in chemistry (20) and a preferred system for exploring the behavior of coupled nonlinear oscillators (21). Our system (Fig. 1 and Fig. S1) consists of a monodisperse emulsion of drops of aqueous BZ solution, whose diameter ranges from 20 to 200  $\mu\text{m}$  dispersed in a continuous phase of oil (17, 18). The drops are surfactant-stabilized to prevent coalescence (22) (*SI Methods*). Chemical coupling between drops is mediated through a small subset of less polar intermediates; primarily an inhibitory component, bromine ( $\text{Br}_2$ ), and to a lesser degree, two excitatory components, bromine dioxide ( $\text{BrO}_2$ ) and bromous acid ( $\text{HBrO}_2$ ),

## Significance

**Turing proposed that intercellular reaction-diffusion of molecules is responsible for morphogenesis. The impact of this paradigm has been profound. We exploit an abiological experimental system of emulsion drops containing the Belousov–Zhabotinsky reactants ideally suited to test Turing's theory. Our experiments verify Turing's thesis of the chemical basis of morphogenesis and reveal a pattern, not previously predicted by theory, which we explain by extending Turing's model to include heterogeneity. Quantitative experimental results obtained using this artificial cellular system establish the strengths and weaknesses of the Turing model, applicable to biology and materials science alike, and pinpoint which directions are required for improvement.**

Author contributions: N.T., N.L., C.G., M.H., I.R.E., and S.F. designed research; N.T., N.L., C.G., G.B.E., and S.F. performed research; N.T., N.L., C.G., M.H., and S.F. analyzed data; and N.T., G.B.E., I.R.E., and S.F. wrote the paper.

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separating cells as chemically specific barriers to diffusion, ignoring any chemical reaction or accumulation of chemicals in the membrane. Turing's resulting reaction-diffusion model consists of a ring of point cells diffusively connected directly to nearest neighbors, expressed as a set of equations each of the following form (1):

$$\frac{d\mathbf{c}_i}{dt} = \mathbf{F}_c(\mathbf{c}_i) + \mathbf{M}_c(\mathbf{c}_{i-1} + \mathbf{c}_{i+1} - 2\mathbf{c}_i), \quad [1]$$

where  $\mathbf{c}_i$  is a vector containing the concentrations of the species in the  $i$ th cell,  $\mathbf{F}_c$  is a vector function describing the kinetics of the  $c$ -species, and  $\mathbf{M}_c$  is a diagonal matrix containing the coefficients of diffusive transport ( $\mu_c$ ) of the  $c$ -species between drops. We describe the chemical kinetics,  $\mathbf{F}_c$ , of the BZ chemistry with a model developed by Vanag and Epstein (26, 27) (VE model, *SI Methods*) that considers four concentrations to vary in time: the inhibitory components bromine ( $\text{Br}_2$ ) and bromide ( $\text{Br}^-$ ), the oxidized form of the catalyst (ferriin), and the activator bromous acid ( $\text{HBrO}_2$ ). The four VE reaction rates,  $\mathbf{F}_c$ , contain the aforementioned four variable chemical species, four more chemicals, whose concentrations are approximated as constants in the model, and nine known rate constants;  $\mathbf{F}_c$  has zero adjustable parameters. Turing did not specify how the coupling strength  $\mu_c$  varies with the physical-chemical parameters. To compare theory with experiment, we supplement the Turing model by explicitly calculating the coupling strength between drops in a capillary using Turing's assumptions noted above, with the caveat that different results arise depending on the assumptions used to produce a geometric point model (27, 28):

$$\mu_c = \frac{2D_c P_c (b+d)}{d^2(a+b)} \left( \ln\left(\frac{b+d}{b}\right) + \frac{a-d}{b+d} \ln\left(\frac{a-d}{a}\right) \right). \quad [2]$$

See Fig. S1 and *SI Methods* for details of the calculation.  $D_c$  and  $P_c$  are the diffusion and partition coefficients of the  $c$ -species in the oil,  $a$  is the length of the BZ drop,  $b$  is the length of the oil gap separating drops, and  $d$  is the diameter of the capillary. The only parameter not measured in Eq. 2 is the partition coefficient of  $\text{HBrO}_2$ ,  $P_x$ .

To elucidate this model, Turing (1) used linear stability analysis (LSA) and identified six possible chemical structures in rings of diffusively coupled identical cells. In LSA, one characterizes how the steady-state concentrations, i.e., those for which  $dc_i/dt = 0$ , respond to small perturbations. If all perturbations decay, then the system is in a stable steady state. However, if any perturbations grow with time, the steady state is unstable, and the fastest growing perturbation is labeled a Turing instability. For a ring of  $N$  cells, the requirement of periodicity restricts dimensionless wavevectors of the perturbations to take on one of three possible values:  $q_{\min} = 0$ ,  $q_{\max} = 2\pi s_{\max}/N$ , where for even numbered rings  $s_{\max} = N/2$  and for odd rings  $s_{\max} = (N-1)/2$ , and  $q = 2\pi s/N$ , where the integer  $s$  ranges from  $0 < s < s_{\max}$ . For each possible  $q$ , the perturbation growth can be either oscillatory with frequency  $\omega > 0$ , or non-oscillatory with  $\omega = 0$ , giving a total of six possible instabilities. Following Turing's nomenclature, the six instabilities ( $a$ - $f$ ) are each characterized by a wavevector and frequency,  $(q, \omega)$ , as follows: Turing case ( $a$ ),  $(q_{\min}, 0)$ ; ( $b$ ),  $(q_{\min}, \omega)$ ; ( $c$ ),  $(q_{\max}, 0)$ ; ( $d$ ),  $(q, 0)$ ; ( $e$ ),  $(q, \omega)$ ; and ( $f$ ),  $(q_{\max}, \omega)$ .

The Turing model, by which we mean the nonlinear rate and coupling equations (Eq. 1), incorporates two significant and untested approximations: considering each cell as a point and simplification of chemical transport by elimination of explicit consideration of the intracellular medium (the oil in our experiments). Furthermore, the use of LSA introduces an additional, severe approximation. The power of the Turing model is that it provides unambiguous physical mechanisms to explain chemical dynamics and morphogenesis. However, as noted by Turing (1),

"This model will be a simplification and an idealization, and consequently a falsification." This raises the question, to what degree do the Turing model and its LSA describe experiment? The answer to this question is of importance to the broad field of reaction-diffusion systems, as over a thousand papers have been published that have built upon the Turing model, which, before this work, has not been experimentally tested for networks of diffusively coupled cells (4). Here, we address six fundamental questions facing reaction-diffusion systems in general: (i) How well does the simplified coupling term,  $\mu_c$ , agree with experiment? (ii) Are there more or fewer than the six predicted Turing linear instabilities? (iii) How are the linear instabilities modified by nonlinearities? (iv) Does the Turing model provide quantitative and predictive understanding of experiment? (v) How do chemical patterns depend on the dimensionality? (vi) Do cells sequentially undergo chemical and then physical morphogenesis?

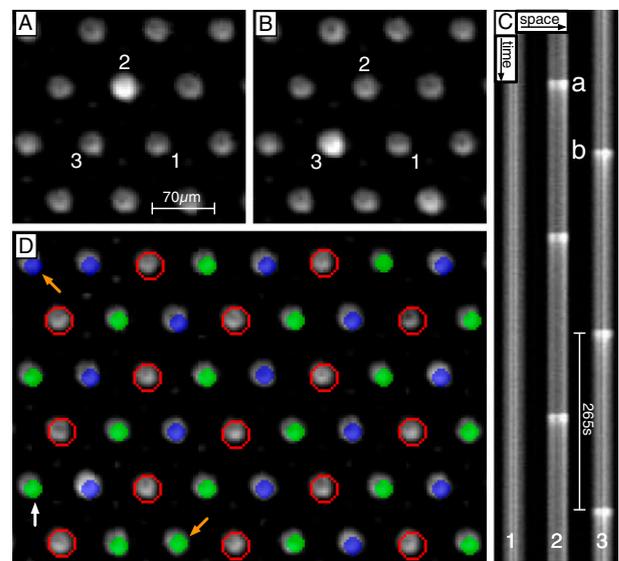
## Results

As a first experimental test of the Turing model, Eq. 1, for cells, we measured the synchronization dynamics of two weakly inhibitory coupled drops at moderate MA concentrations, where uncoupled drops oscillate and bromine can be considered as the sole intercell transporter (18), i.e.,  $P_x = 0$ . We filled cylindrical capillaries with drops, used light both to chemically isolate a pair of adjacent drops and to set the initial phase difference between the isolated drops, and measured the phase difference between the two drops as a function of time, as shown in Fig. S1. Viewed in transmission, the oxidized state appears bright, whereas the reduced state appears dark. Ultimately, the drops synchronize with a phase difference of  $\pi$  radians (18, 29). Fig. S1 and Movie S1 present experimental synchronization rates as a function of drop sizes (50–200  $\mu\text{m}$ ) and oil gaps (10–200  $\mu\text{m}$ ) for  $\sim 100$  drop pairs; for these conditions, rates varied by a factor of 30. Excellent fits were obtained between the experimentally measured synchronization rates and the full nonlinear solution of Eq. 1 if we treated  $\mu_c$  as a fitting parameter, which varied for each drop size and oil gap. We also fit synchronization rates using the explicitly calculated coupling strength, Eq. 2. Although the functional form of the coupling strength (Eq. 2) fit the time-dependent synchronization data well for a wide range of oil gaps and drop diameters, the combination of the Turing model (Eq. 1) with our explicit calculation of the interdrop coupling (Eq. 2), overestimates the coupling strength by nearly two orders of magnitude. That is, we replaced  $\mu_c$  of Eq. 2 with  $f\mu_c$ , and although theory predicts  $f = 1$ , experimentally we find  $f = 0.0152$ . Despite this discrepancy, the fact that only one phenomenological parameter,  $f$ , is needed to reconcile theory and experiment over a wide range of coupling strengths is an improvement over the original Turing model, where a different phenomenological parameter,  $\mu_c$ , is fitted for each drop diameter and oil gap. As a guide to theorists motivated to improve our calculation of the coupling strength, we note that the Turing model assumes a vanishing thin membrane, whereas our experimental system has a finite-sized oil gap. Our model of coupling strength, based on the assumptions of the Turing model, neglects four factors, each of which reduces coupling: nonuniformity in chemical gradients in the drop and the oil, accumulation of chemicals in the oil, time taken to diffuse across the oil, and chemical reactions within the oil.

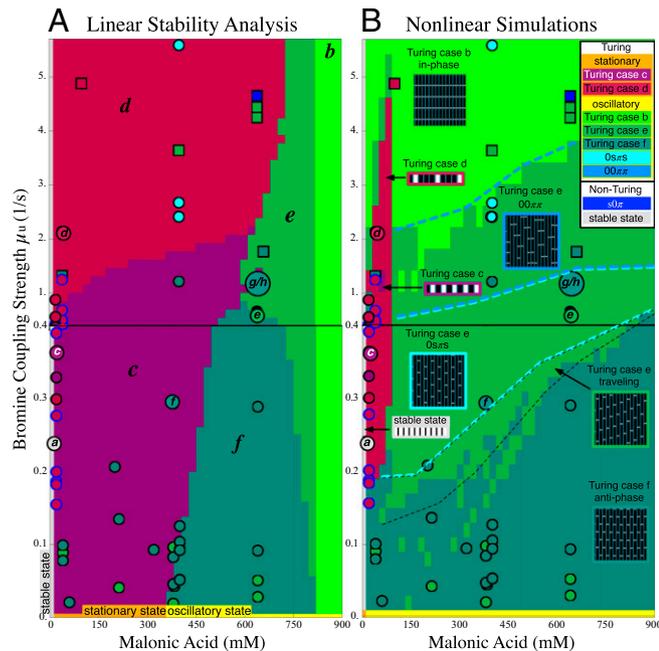
Having established the Turing model is quantitatively valid for a wide range of synchronization conditions with a single experimentally determined constant in the coupling term,  $f = 0.0152$ , we prepared a series of one-dimensional arrays of drops in rings and lines and determined the long-term emergent chemical states as a function of the two variables that most strongly control interdrop behavior: MA concentration and coupling strength. Coupling strength,  $\mu_c$ , is conveniently tuned experimentally by varying the drop size  $a$  and oil gap  $b$  using microfluidics. In Fig. 1  $A$ - $F$ , we illustrate examples of six distinct patterns with

the symmetry corresponding to Turing cases (a–f). Five of the six patterns, Fig. 1 A and C–F, appear where predicted by theory in Fig. 2B. Four of the six patterns are identified with Turing cases (c, d, e, f). The fifth pattern has the same symmetry as Turing case (a). However, this pattern is predicted to be a stationary, stable state for MA concentrations below 1 mM and therefore does not arise from a Turing instability, underscoring the point that observation of a chemical state with a pattern corresponding to a Turing instability is insufficient evidence to prove the state arises from a Turing mechanism (3, 6, 8, 30). Notably, the pattern with the symmetry of Turing case (b) is observed in a region of parameter space not predicted by theory. This is the sole discrepancy between theory and experiment, and we suspect that it reflects a shortcoming of the VE model. See *SI Discussion* for expanded analysis of each case and *Movies S2* and *S3* of the spatial-temporal patterns.

The behavior of finite rings depends on the number of drops,  $N$ , in the ring as seen in Fig. 1 G and H and *Movie S3* for two rings with identical chemical composition, drop size, and spacing, but with five and six drops, respectively. For these particular chemical conditions and for  $N$  even, LSA predicts antiphase oscillations, corresponding to Turing instability (f) characterized by the wavevector–frequency pair ( $q_{\max}, \omega$ ), as defined previously and in more detail in *SI Text*. Turing’s prediction is that for  $N$  odd, no two drops will undergo an oxidation transition simultaneously; there will be  $N$  beats per measure, whereas for  $N$  even there will be two beats per measure. For a ring of five drops,



**Fig. 3.** Observations of 2D arrays of  $s0\pi$  states. (A and B) Single frames demonstrating the  $s0\pi$  state. The frames correspond to the first two oxidation transitions, labeled a and b in C. (C) Space–time plot of drops 1–3 shown in A and B. Drop 1 is stationary, whereas drops 2–3 remain oscillatory with a phase difference of  $\pi$ . (D) A combined image where the stationary drops are outlined in red and the oscillatory drops are color coded by their phase difference,  $\phi_i = \theta_i - \theta_{\text{ref}}$ , where  $0 \leq \phi \leq \pi$  and  $\theta_{\text{ref}}$  is the phase of the drop indicated with the white vertical arrow. Drops where  $\phi_i < \pi/2$  are green and  $\phi_i > \pi/2$  are blue. Notice that every third drop is stationary, and every oscillatory drop is out of phase with its immediate neighbors; two exceptions are noted with orange arrows. See *Movie S6*. Chemical conditions: 300 mM bromate, 3 mM ferrioin, 0.4 mM Ruppy, 80 mM acid, 640 mM MA, and 10 mM NaBr. Drop size is  $\sim 70 \mu\text{m}$ .



**Fig. 2.** (A) Linear stability analysis (LSA) and (B) nonlinear simulations (NLS) of the Vanag–Epstein BZ model, as a function of bromine coupling strength,  $\mu_c$ , and MA concentration. Note the scale change on the vertical axis at  $\mu_c = 0.4$ . In both figures, red hues represent stationary states, and green hues, oscillatory states. The data are plotted with circles for 1D experiments and squares for 2D experiments. The red disks outlined in blue indicate a transient initial cluster state settling into a stationary Turing instability. The data points with a letter in the center are included in Fig. 1. Space–time plots of oscillatory states are *Inset* in square areas; stationary states are *Inset* in rectangular areas. The dashed boundaries within the region of Turing case (e) of the NLS diagram indicate location of traveling waves, the  $0s\pi\pi$  state (Fig. S6), and the  $00\pi\pi$  cluster state. All experimental data points are plotted according to the definitions of  $\mu$  given in the text with  $f = 0.14$ . (See *SI Discussion* for more on the plotting of data points.)

LSA predicts a waveform  $C_5(r, t) \propto \exp(i(4\pi r/5 - \omega t))$ , with  $r \in (0, 1, 2, 3, 4)$  the drop number. In this expression, the phase is chosen such that a drop is oxidized when  $4\pi r/5 - \omega t$  is equal to a multiple of  $2\pi$ . As time advances in increments of one-fifth of a period, the oxidized state in Fig. 1G moves along the ring in a pentagramal sequence from drops  $0 \rightarrow 3 \rightarrow 1 \rightarrow 4 \rightarrow 2$ . For the ring of six drops,  $C_6(r, t) \propto \exp(i(\pi r - \omega t))$ , with  $r \in (0, 1, 2, 3, 4, 5)$ . As shown in Fig. 1H, all even-numbered drops oxidize simultaneously at the beginning of a period, and one-half a period later, all odd-numbered drops oxidize. In Fig. S4 and *SI Methods*, we present the LSA predictions for rings with three, four, five, and six drops and the corresponding experiments. For rings of drops, Turing’s LSA theory and our experiments are in complete agreement.

Using published chemical rate constants of the VE model (26) (Eq. S2 and Table S2), we calculate two state diagrams, one using Turing’s LSA and the other the full nonlinear simulation (NLS) of equations (Eq. 1) in one dimension, shown in Fig. 2. These theory plots have no adjustable parameters, as the Turing model treats the coupling strength,  $\mu_c$ , as an independent variable. However, to assign coupling strengths to experiment, we explicitly calculate coupling strength using Eq. 2 to which we introduce a fitting parameter by replacing  $\mu_c$  of Eq. 2 with  $f\mu_c$ . We also fit the partition coefficient of the activator,  $P_x$ . The best agreement between the NLS and experiment was obtained for  $P_x = 0.05$  and  $f = 0.14$ . With respect to the experimental state diagram, the NLS overestimates coupling strength eightfold, which is the same trend as in the case of the synchronization experiments. In experiments and in the NLS at low MA, we find a stable stationary state in which all of the drops are in-phase. This has the same pattern as Turing state (a), but as noted previously, LSA reveals this state is stable and it cannot be considered



The drops, produced microfluidically as spheres, are stored in a rectangular capillary and are deformed into cylindrical disks with the same height as the capillary. The intensity of each drop is a monotonic function of the fraction of oxidized BZ catalyst it contains. As shown in Fig. 4A, the drops are initially homogeneous in chemistry and drop size. After an initial induction time, the drops undergo a transition from this unstable steady state to Turing case (*d*), in which one out of three drops is in the reduced (dark) state and two out of three are oxidized (bright), shown in Fig. 4C. This chemical differentiation occurs with the drop size remaining constant. The oxidized drops consume reagents faster than the reduced drops. This creates an osmotic pressure imbalance, causing water to flow from the oxidized to reduced drops, creating a morphological transformation in which the initially homogeneous cells differentiate into two populations with distinct chemical redox states and physical sizes, as shown in Fig. 4E. Quantitative measurements of the volume changes are in agreement with theory, as elaborated in *SI Text*.

## Conclusion

Turing's model predicts the circumstances under which initially homogeneous diffusively coupled cells will spontaneously evolve spatiotemporal chemical structures. However, only a small subset of chemical reactions lead to the Turing instabilities; most reactions remain stably homogeneous. In emulsions of the oscillatory BZ chemical reaction, tuning coupling strength and chemical dynamics by changing drop size and MA concentration, respectively, reveals seven distinct chemical structures, six of which were predicted by theory. Turing's model eliminates the

oil phase separating cells and treats the coupling strength as a free parameter. We extended Turing's model to explicitly calculate the coupling strength. Experiments revealed that the extended model overestimated intercellular coupling by nearly two orders of magnitude. One experimentally determined parameter was introduced to reconcile theory and experiment for a wide range of conditions; eliminating this one phenomenological parameter remains a theoretical challenge. LSA of the Turing model captures most of the qualitative features of the observed chemical pattern formation, thereby providing a mechanistic explanation of pattern selection. However, the full nonlinear model must be solved to achieve quantitative agreement between experiment and theory. We observe one chemical pattern inconsistent with the original Turing model and propose a generic mechanism whereby slight heterogeneity in the cells leads to a state of mixed dynamical and stationary character. The Turing model is regarded as a metaphor for morphogenesis in biology, useful for a conceptual framework and to guide modeling, but not for prediction (7). In contrast, in this chemical system, we demonstrated that the Turing model quantitatively explains "materials morphogenesis" in which cellular compartments first chemically and then physically differentiate, raising the possibility of exploiting this form of reaction-diffusion chemistry for materials science applications.

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