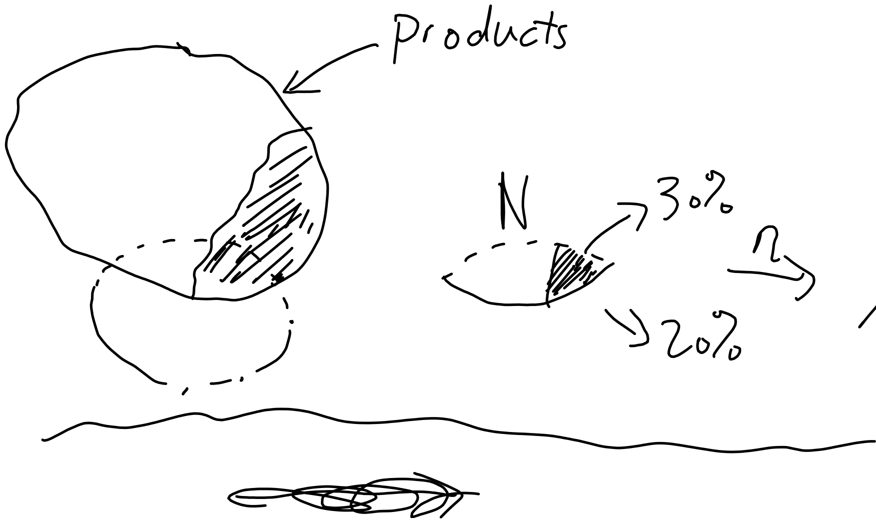
Estimation of proportion



n/N are good $\longrightarrow$ "true" proportion in the whole population.

$$P(\cancel{\text{Data}}\ \text{Data} \mid \text{parameter})$$

$$P(\text{parameter} \mid \text{Data}) \checkmark$$

$$\star\quad P(\text{parameter} \mid \text{Data}) = \frac{P(\text{Data} \mid \cancel{\text{Data}}\ \text{parameter}) \cdot P(\text{parameter})}{\int P(\text{Data} \mid \text{parameter})\, P(\text{parameter})\, d\,\text{parameter}}.$$

Friday, September 18, 2020 10:40 AM

$$P(f|n,N) = \frac{P(n,N|f)\,P(f)}{P(n,N)}$$

f : ~~pro~~ real, true proportion in the total population

Infer.

$$P(n,N|f) = \binom{N}{n} f^n (1-f)^{N-n} \quad \text{(Binomial.)}$$

f : immuned individuals (vaccine works) (good products)

$$P(n,N) = \int_0^1 P(n,N|f')\,P(f')\,df'$$

$$= \int_0^1 \binom{N}{n} f'^n (1-f')^{N-n}\, p(f')\,df'$$

$\boxed{p(f) \equiv 1}$ prior

$$P(f|n,N) = \frac{\binom{N}{n} f^n (1-f)^{N-n}}{\int_0^1 \binom{N}{n} f'^n (1-f')^{N-n}\,df'} = \frac{f^n (1-f)^{N-n}}{\int_0^1 g^n (1-g)^{N-n}\,dg} = \text{Posterior distribution of } f$$

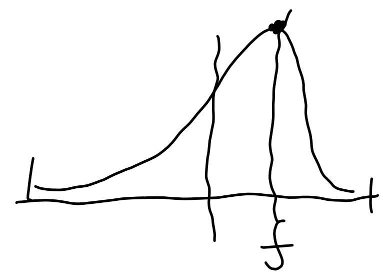
Friday, September 18, 2020 10:49 AM

posterior

$$h(f) \sim \frac{f^n (1-f)^{N-n}}{\int_0^1 g^n (1-g)^{N-n}\, dg}$$

$E(f) \sim$ mean of ~~h(f)~~ f.

Mode(f) $\sim$ highest point of $h(f)$



$$\ln h(f) \sim n \ln f + (N-n) \ln(1-f) - \ln(\text{Denominator}).$$

$\ln a^b = b \cdot \ln a$

$(\ln a)' = \frac{1}{a}$

$$\frac{\partial \ln h(f)}{\partial f} \sim \frac{n}{f} + \frac{N-n}{1-f} \cdot (-1) = 0$$

$$\frac{n}{f} = \frac{N-n}{1-f} \Rightarrow n(1-f) = f(N-n) \Rightarrow f = \frac{n}{N}$$

Friday, September 18, 2020 11:06 AM

$$\int_0^1 g^n (1-g)^{N-n} \, dg = \frac{n!\,(N-n)!}{(N+1)!}$$

$n! \sim n(n-1)\ldots 1$

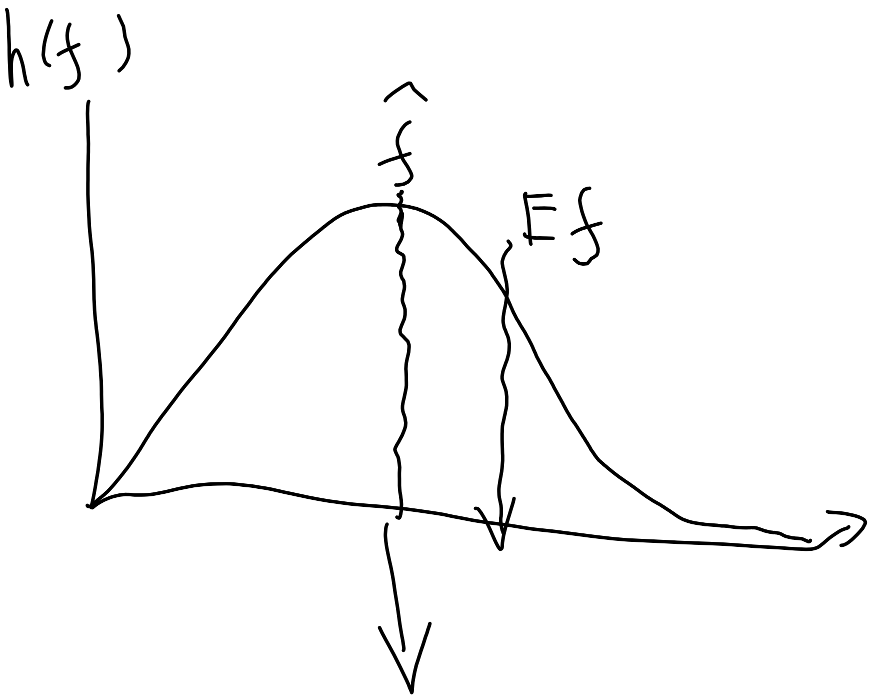
$$h(f) = \frac{(N+1)!}{n!\,(N-n)!} f^n (1-f)^{N-n}$$ → Proper. Posterior distribution

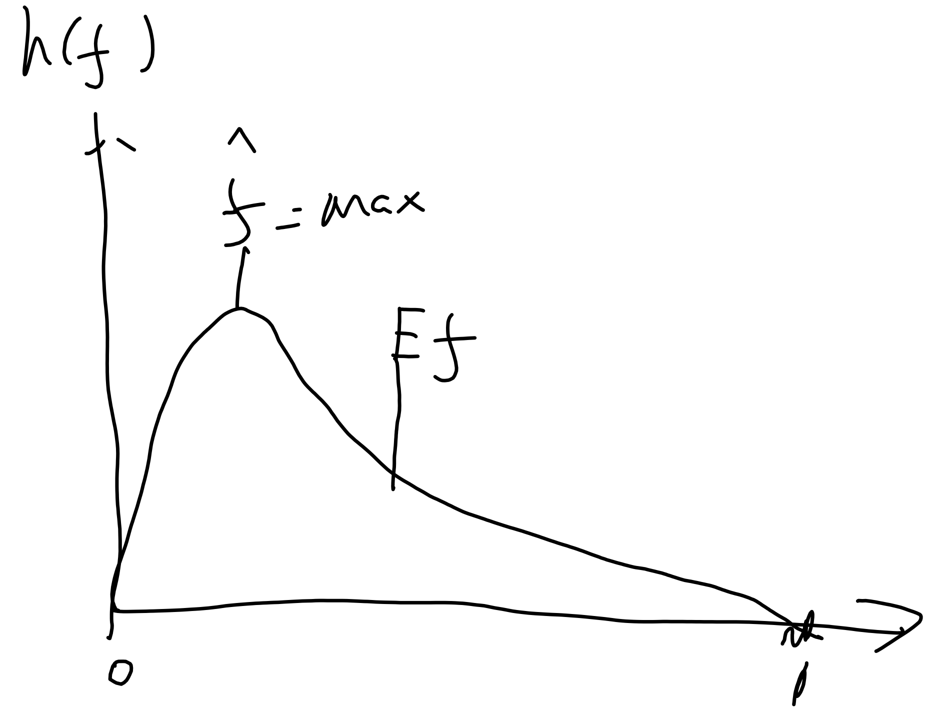
$h(f)$ maximized at $f = \frac{n}{N}$

$\frac{n+1}{N}$

$h(f) \sim P(f \mid \text{Data}) <$

Friday, September 18, 2020 11:25 AM





Run the test another time 1 ✓ 0 X.

$$\int_0^1 f\, h(f)\, df = \frac{n+1}{N+2}$$

$$\text{Mode} = \frac{n}{N}$$