Information Content

event – p $\qquad I = \log \frac{1}{p}$ (Shannon Information)

N possible outcomes for event $\qquad I = \log \frac{1}{1/N} = \log N \qquad N\uparrow\ I\uparrow$

"how surprised I am"

"How much I learned by seeing the actual outcome"

Entropy

$P(x)$ $\qquad H = \left\langle \log \frac{1}{P(x)} \right\rangle = \int_x P(x) \log \frac{1}{P(x)} dx$

$$= - \int_x P(x) \log P(x)\, dx$$

the average information

$D_{KL} = \int P(x) \log \frac{P(x)}{g(x)} \geq 0$ divergence or "distance" between two distributions

$$MI(X,Y) = \int_{x,y} P(x,y) \log \frac{P(x,y)}{P(x)P(y)} \geq 0$$

The principle of maximum entropy

(E.T.)
Ed Jaynes
1957

"the only unbiased assignment"

"maximally noncommittal with regard to missing information"

"no possibility is ignored"

"It assigns positive weights to every situation that is not absolutely excluded by the given information"

Graduate program has 3 labs

P_1 = prob. students join lab 1

P_2 = " " " lab 2

P_3 = " " " lab 3

P_1, P_2, P_3? according to MAXENT?

$$H = -P_1 \log P_1 - P_2 \log P_2 - P_3 \log P_3 - \lambda (P_1 + P_2 + P_3 - 1)$$

$$\frac{\delta H}{\delta P_1} = -\log P_1 - P_1 \cdot \frac{1}{P_1} - \lambda = 0 \rightarrow \log P_1 = -(1+\lambda)$$

$$\log P_2 = -(1+\lambda)$$

$$\frac{\delta}{\delta P_2} = -\log P_2 - 1 - \lambda$$

$$\log P_3 = -(1+\lambda)$$

After 1 week, you have more data

	# year to phD
P_1	4
P_2	5
P_3	6

on average students take a total of 5.7 year to Finish

$\longrightarrow$ BREAKROOMS

$$L = -P_1 \log P_1 - P_2 \log P_2 - P_3 \log P_3 - \lambda(P_1 + P_2 + P_3)$$
$$- \alpha (4 \cdot P_1 + 5 \cdot P_2 + 6 \cdot P_3 - 5.7$$

$$\frac{\delta L}{\delta P_1} = -\log P_1 - 1 - \lambda - 4\alpha = 0 \longrightarrow P_1 = e^{-(1+\lambda)} e^{-4\alpha}$$

$$\frac{\delta L}{\delta P_2} = -\log P_2 - 1 - \lambda - 5\alpha \longrightarrow P_2 = e^{-(1+\lambda)} e^{-5\alpha}$$

$$\frac{\delta L}{\delta P_3} = -\log P_3 - 1 - \lambda - 6\alpha \longrightarrow P_3 = e^{-(1+\lambda)} e^{-6\alpha}$$

$$1 = e^{-(1+\lambda)} \left[e^{-4\alpha} + e^{-5\alpha} + e^{-6\alpha} \right]$$

$$P_1 = \frac{e^{-4\alpha}}{e^{-4\alpha} + e^{-5\alpha} + e^{-6\alpha}}$$

$$P_2 = \frac{e^{-5\alpha}}{e^{-4\alpha} + e^{-5\alpha} + e^{-6\alpha}}$$

$$P_3 = \frac{e^{-6\alpha}}{e^{-4\alpha} + e^{-5\alpha} + e^{-6\alpha}}$$

$$4e^{-4\alpha} + 5e^{-5\alpha} + 6e^{-6\alpha} = 5.7\left(e^{-4\alpha} + e^{-5\alpha} + e^{-6\alpha}\right)$$

$$1.7e^{-4\alpha} + 0.7e^{-5\alpha} - 0.3e^{-6\alpha} = 0 \longrightarrow \alpha\ ?$$

$\alpha = -1.34$

$P_1 = 0.05$

$P_2 = 0.20$

$P_3 = 0.75$

.

After a month

p	t	a (articles)
P_1	$t_1 = 4$	$a_1 = 1$
P_2	$t_2 = 5$	$a_2 = 2$
P_3	$t_3 = 6$	$a_3 = 3$

$\langle t \rangle = 5.7 \qquad \langle a \rangle = 1.5$

$$L = -P_1 \log P_1 - P_2 \log P_2 - P_3 \log P_3 - \lambda (P_1 + P_2 + P_3)$$
$$- \alpha (4P_1 + 5P_2 + 6P_3 - 5.7)$$
$$- \beta (P_1 + 2P_2 + 3P_3 - 1.5)$$

$$\frac{\delta L}{\delta P_1} = -\log P_1 - 1 - \lambda - 4\alpha - \beta = 0$$

$$\frac{\delta L}{\delta P_2} = -\log P_2 - 1 - \lambda - 5\alpha - 2\beta = 0$$

$$\frac{\delta L}{\delta P_3} = -\log P_3 - 1 - \lambda - 6\alpha - 3\beta = 0$$

$$P_1 = \frac{e^{-4\alpha-\beta}}{e^{-4\alpha-\beta} + e^{-5\alpha-2\beta} + e^{-6\alpha-3\beta}}$$

$$P_2 = \frac{e^{-5\alpha-2\beta}}{Z}$$

$$P_3 = \frac{e^{-6\alpha-3\beta}}{Z}$$

$$4e^{-4\alpha-\beta} + 5e^{-5\alpha-2\beta} + 6e^{-6\alpha-3\beta} = 5.7Z$$

$$e^{-4\alpha-\beta} + 2e^{-5\alpha-2\beta} + 3e^{-6\alpha-3\beta} = 1.5Z$$

$$\boxed{\left\{\begin{aligned} &1.7e^{-4\alpha-\beta} + 0.7e^{-5\alpha-2\beta} - 0.3e^{-6\alpha-3\beta} = 0 \\ &0.5e^{-4\alpha-\beta} - 0.5e^{-5\alpha-2\beta} - 1.5e^{-6\alpha-3\beta} = 0 \end{aligned}\right.}$$