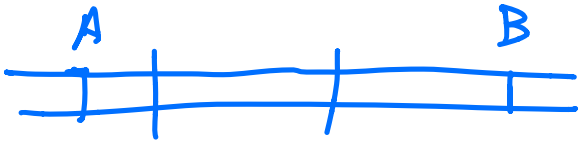
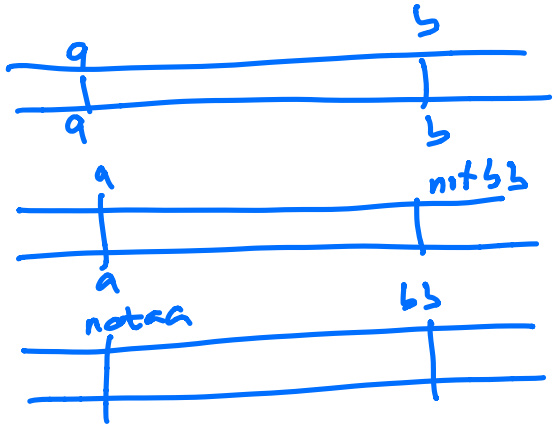




$P(blue \mid aa \wedge bb) = 1.0$

$P(blue \mid aa \wedge \overline{bb}) = 0.8$

$P(blue \mid \overline{aa} \wedge bb) = 0.9$

$$P(\text{blue}) = P(\text{blue} \mid aa \wedge bb)\, P(aa \wedge bb)$$
$$+ P(\text{blue} \mid aa \wedge \bar{bb})\, P(aa \wedge \bar{bb})$$
$$+ P(\text{blue} \mid \bar{aa} \wedge bb)\, P(\bar{aa} \wedge bb)$$

$\bar{aa} = \text{not}(aa)$
$\bar{bb} = \text{not}(bb)$

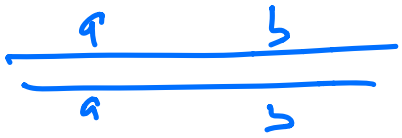
i) if aa and bb are independent

$$P(aa \wedge bb) = P(aa)\, P(bb)$$
$$P(\bar{aa} \wedge bb) = P(\bar{aa})\, P(bb) = [1 - P(aa)]\, P(bb)$$
$$P(aa \wedge \bar{bb}) = P(aa) \cdot P(\bar{bb}) = P(aa)\,[1 - P(bb)]$$

assume
$P(aa) = 0.1$
$P(bb) = 0.01$

$$P(\text{blue}) = 1.0 \times 0.1 \times 0.01 + 0.8 \times 0.9 \times 0.01 + 0.9 \times 0.1 \times 0.99$$
$$= 0.001 + 0.0072 + 0.0891 = 0.0973$$

ii) if aa and bb always occurr together



$$P(aa \wedge bb) = P(aa \mid bb) \cdot P(bb) = P(bb)$$
$$= P(bb \mid aa)\ P(aa) = P(aa)$$
$$P(aa \wedge bb) = P(aa) = P(bb) \quad (\text{assume} = 0.1)$$

$$P(aa \wedge \overline{bb}) = P(aa \mid \overline{bb})\ P(\overline{bb}) = 0$$

$$P(\overline{aa} \wedge bb) = P(\overline{aa} \mid bb) \cdot P(bb) = 0$$

$$P(blue) = P(blue \mid aa \wedge bb) \cdot P(aa)$$
$$= 1.0 \times 0.1 = 0.1$$

Francis pope.

H - human
A - Alien

$$P(F|H) = \frac{1}{7 \cdot 10^9}$$

$$P(H|F) = \frac{P(F|H) \cdot P(H)}{P(F)}$$

$$= \frac{P(F|H) \cdot P(H)}{P(F|H) \cdot P(H) + P(F|A) \cdot P(A)} \approx 1$$

$P(F|A)$ very small, $P(A)$ very small

$P(\text{Innocent}|+)$ in a population of $N = 10$ millions

$$P(I|+) = \frac{P(+|I)\,P(I)}{P(+|I)\,P(I) + P(+|G)\cdot P(G)}$$

$P(G) = \frac{1}{10^9} = 10^{-7}$ chance that a random person is guilty

$P(I) = 1 - P(G) \simeq 1$ " " " " is innocent

$P(+|G) = 1$ [no false negatives]

$$P(I|+) = \frac{10^{-6}}{10^{-6} + 10^{-7}} = \frac{1}{1+0.1} = \frac{1}{1.1} = 0.909$$

$>$ 90% chance that a + test is innocent !!